Journal of Engineering Physics and Thermophysics, Vol. 73, No. 3, 2000

COMPARISON OF THE EFFICIENCIES OF RECTIFICATION TRAYS IN COUNTERCURRENT MOTION OF PHASES WITH ALLOWANCE FOR LIQUID MIXING

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UDC 66.048.375

Consideration is given to a tray liquid in the intermediate state between ideal displacement and total mixing, for which purpose the degree of attaining the latter state by the liquid is introduced. The composition of the vapor after leaving an ideal tray is assumed to be in equilibrium with that of the liquid in its intermediate state. Four variants of interrelation of the real and ideal trays are considered; the efficiencies for the vapor and the liquid are proved to be equal for each of them. Ratios for the efficiencies of all the variants are obtained. A comparison is made of the efficiency ratios as functions of liquid mixing.

Mixing of a tray liquid exerts a pronounced effect on the efficiency of mass transfer. In the literature, borderline states, namely, total mixing of a tray liquid and its ideal displacement, are described in detail. In the first case, the composition of the tray liquid is the same everywhere on the tray and is equal to the composition of the liquid leaving the tray (Fig. 1a). For an ideal tray, a vapor is in equilibrium with a liquid of this composition:

$$(y_n^*)_{\mathrm{m}} = m x_{n-1}^* . \tag{1}$$

In the second case, liquid mixing is absent and its composition gradually changes from the initial to the final concentration. The vapor also becomes gradually enriched with a highly volatile component and on leaving the tray it is in equilibrium, in the case of countercurrent motion of phases, with an oncoming liquid (Fig. 1b):

$$(y_n^*)_{\mathrm{r}} = m x_n^* . \tag{2}$$

In practice, most often there occurs a certain intermediate state, which is characterized by partial mixing of a liquid. It can be represented by models in which the degree of mixing is determined as a function of a number of design and technological factors [1, 2]. In the cellular model, the degree of liquid mixing is allowed for by the number of total-mixing cells with ideal displacement of the liquid between the cells. In the circulation model, it is assumed that some amount of a running-down liquid returns to the tray entry. In the diffusional model, the degree of liquid mixing is characterized by a dimensionless Péclet number.

Let us assume that some amount of a tray liquid is totally mixed and its other part moves in the regime of ideal displacement. We assign φ of the totally mixed liquid on the ideal tray, which is characterized by equilibrium condition (1). Then the amount of the unmixed liquid, for which expression (2) holds, is $1 - \varphi$. In total mixing of the entire liquid, $\varphi = 1$; in its ideal displacement, $\varphi = 0$. In this case, the composition of the vapor after leaving the ideal tray can be represented as a function of the amount of the mixed liquid by the formula

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Belarusian State Technological University, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 73, No. 3, pp. 514-519, May–June, 2000. Original article submitted April 9, 1999.

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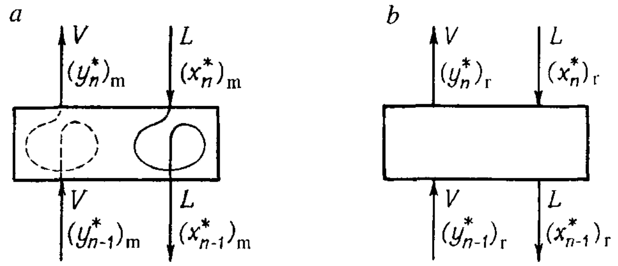

Fig. 1. Mass transfer schemes: a) total mixing; b) ideal displacement.

$$y_n^* = \varphi\,(y_n^*)_m + (1-\varphi)\,(y_n^*)_r = \varphi m x_{n-1}^* + (1-\varphi)\,m x_n^* \,. \tag{3}$$

An analysis of the interrelation of the real and ideal trays reveals that four variants are possible [3]. The conditions common for all of them are constancy of the molar vapor and liquid flows across both trays and equality of heat transfer between both trays and the surrounding medium.

In the first variant, the compositions of the oncoming vapor flows and the leaving liquid flow coincide for the ideal and real trays, which is characteristic of the Murphree model in analyzing the efficiency in the vapor phase [4, 5]:

$$(y_{n-1}^*)_1 = y_{n-1}\,;\quad (x_{n-1}^*)_1 = x_{n-1}\,,$$

and the equations of material balance for the real and ideal trays can be written, respectively, in the form

$$L\,(x_n - x_{n-1}) = V\,(y_n - y_{n-1})\,, \tag{4}$$

$$L\,[(x_n^*)_1 - x_{n-1}] = V\,[(y_n^*)_1 - y_{n-1}]\,. \tag{5}$$

In this case, the tray efficiency for the vapor and liquid phases can be represented by the corresponding relations

$$E_{1\varphi,\mathrm{v}} = \frac{y_n - y_{n-1}}{(y_n^*)_1 - y_{n-1}}\,, \tag{6}$$

$$E_{1\varphi,\mathrm{liq}} = \frac{x_n - x_{n-1}}{(x_n^*)_1 - x_{n-1}}\,. \tag{7}$$

Equilibrium condition (3) for the first variant acquires the form

$$(y_n^*)_1 = \varphi m x_{n-1} + (1-\varphi)\,m\,(x_n^*)_1\,. \tag{8}$$

On solving simultaneously expressions (4)-(6) and (8) as well as (4), (5), (7), and (8), we can arrive at the equation

$$x_n - x_{n-1} = \frac{E_{1\varphi,\mathrm{v}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV} - 1 + \varphi} = \frac{E_{1\varphi,\mathrm{liq}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV} - 1 + \varphi}\,, \tag{9}$$

from which the equality of the efficiences in the vapor and liquid phases follows; this allows us to write

$$E_{1\varphi} = E_{1\varphi,\mathrm{v}} = E_{1\varphi,\mathrm{liq}} = \frac{y_n - y_{n-1}}{(y_n^*)_1 - y_{n-1}} = \frac{x_n - x_{n-1}}{(x_n^*)_1 - x_{n-1}} . \tag{10}$$

Unlike the aforesaid, in the second variant [3] for the ideal and real trays the compositions of the leaving vapor flows and the oncoming liquid flows coincide. This interrelation exists in the Murphree model in analyzing the efficiency in the liquid [4, 5] and is specified by the expressions

$$(y_n^*)_2 = y_n , \quad (x_n^*)_2 = x_n .$$

In accordance with the specific features mentioned, the equation of material balance of the ideal tray has the form

$$L\,[x_n - (x_{n-1}^*)_2] = V\,[y_n - (y_{n-1}^*)_2] , \tag{11}$$

while for a real tray, it has the form of (4). The tray efficiency for the considered variant in the vapor and liquid phases can be represented by the corresponding relations

$$E_{2\varphi,\mathrm{v}} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_2} , \tag{12}$$

$$E_{2\varphi,\mathrm{liq}} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_2} , \tag{13}$$

while condition (3) can be described by the equation

$$y_n = \varphi m\,(x_{n-1}^*)_2 + (1-\varphi)\,m x_n . \tag{14}$$

Solving simultaneously expressions (4), (11), (12), and (14), on the one hand, and (4), (11), (13), and (14), on the other hand, allows us to obtain the relation

$$x_n - x_{n-1} = \frac{E_{2\varphi,\mathrm{v}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{E_{2\varphi,\mathrm{v}}\left(\dfrac{L}{mV} - 1\right) + \varphi} = \frac{E_{2\varphi,\mathrm{liq}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{E_{2\varphi,\mathrm{liq}}\left(\dfrac{L}{mV} - 1\right) + \varphi} , \tag{15}$$

which shows the equality of the efficiencies in the vapor and liquid phases for the second variant, i.e.,

$$E_{2\varphi} = E_{2\varphi,\mathrm{v}} = E_{2\varphi,\mathrm{liq}} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_2} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_2} . \tag{16}$$

A specific feature of the scheme of change in the concentrations considered in the third variant [3] is the equality of the compositions of the vapor and the liquid entering the ideal and real trays, which is characteristic of the Hausen model [5, 6]:

$$(y_{n-1}^*)_3 = y_{n-1} \, ; \quad (x_n^*)_3 = x_n .$$

In this case, the equation of material balance for the real tray is similar to (4), while for the ideal tray it can be represented in the form

$$L\,[x_n - (x_{n-1}^*)_3] = V\,[(y_n^*)_3 - y_{n-1}]\,, \tag{17}$$

and the efficiency in the vapor and liquid phases is, respectively,

$$E_{3\varphi,\mathrm{v}} = \frac{y_n - y_{n-1}}{(y_n^*)_3 - y_{n-1}}\,, \tag{18}$$

$$E_{3\varphi,\mathrm{liq}} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_3}\,. \tag{19}$$

Formula (3) is specified for this variant by the expression

$$(y_n^*)_3 = \varphi m\,(x_{n-1}^*)_3 + (1-\varphi)\,mx_n\,. \tag{20}$$

As a result of simultaneous solution of expressions (4) and (17)-(20), we arrive at the relation

$$x_n - x_{n-1} = \frac{E_{3\varphi,\mathrm{v}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV} - E_{3\varphi,\mathrm{v}} + \varphi} = \frac{E_{3\varphi,\mathrm{liq}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV} - E_{3\varphi,\mathrm{liq}} + \varphi}\,, \tag{21}$$

from which the equality of the tray efficiencies in the vapor and liquid phases follows for the third variant, i.e.,

$$E_{3\varphi} = E_{3\varphi,\mathrm{v}} = E_{3\varphi,\mathrm{liq}} = \frac{y_n - y_{n-1}}{(y_n^*)_3 - y_{n-1}} = \frac{x_n - x_{n-1}}{x_n - (x_{n-1}^*)_3}\,. \tag{22}$$

Mass transfer in the fourth variant [3] is characterized by the equality of the compositions of the vapor and liquid flows leaving the real and ideal trays:

$$(y_n^*)_4 = y_n\,; \quad (x_{n-1}^*)_4 = x_{n-1}\,.$$

As in the previous variants for the real tray, formula (4) holds in this case. The equation of material balance of the ideal tray can be represented as follows:

$$L\,[(x_n^*)_4 - x_{n-1}] = V\,[y_n - (y_{n-1}^*)_4]\,. \tag{23}$$

In accordance with the assumptions made, the tray efficiency in the vapor and liquid phases can be represented, respectively, as

$$E_{4\varphi,\mathrm{v}} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_4}\,, \tag{24}$$

$$E_{4\varphi,\mathrm{liq}} = \frac{x_n - x_{n-1}}{(x_n^*)_4 - x_{n-1}}\,, \tag{25}$$

while equilibrium condition (3) can be represented as

$$y_n = \varphi m x_{n-1} + (1 - \varphi)\, m\, (x_n^*)_4 \,. \tag{26}$$

A simultaneous solution of expressions (4) and (23)-(26) allows us to obtain the dependence

$$x_n - x_{n-1} = \frac{E_{4\varphi,\mathrm{v}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV}E_{4\varphi,\mathrm{v}} - 1 + \varphi} = \frac{E_{4\varphi,\mathrm{liq}}\left(x_{n-1} - \dfrac{y_{n-1}}{m}\right)}{\dfrac{L}{mV}E_{4\varphi,\mathrm{liq}} - 1 + \varphi}\,, \tag{27}$$

which points to the equality of the efficiencies in the vapor and liquid phases in the fourth variant as well, i.e.,

$$E_{4\varphi} = E_{4\varphi,\mathrm{v}} = E_{4\varphi,\mathrm{liq}} = \frac{y_n - y_{n-1}}{y_n - (y_{n-1}^*)_4} = \frac{x_n - x_{n-1}}{(x_n^*)_4 - x_{n-1}}\,. \tag{28}$$

The analysis performed shows that in each of the considered variants the efficiencies in the vapor phase are equal to those in the liquid phase. A comparison of the efficiencies reveals their coincidence for individual variants. In particular, the efficiency expressions (6) and (18) and (12) and (24) for the vapor phase and (7) and (25) and (13) and (19) for the liquid are identical. However, the efficiencies calculated by the above paired relations are not equal to each other in the general case, since they must be considered in combination with the corresponding equations of material balance and the equilibrium conditions of the vapor and liquid compositions, which are individual for each variant. However, in particular cases, with a certain ratio of the flows and concentrations the values of the mentioned groups of efficiencies can coincide.

The left-hand sides of formulas (9) (15), (21), and (27) display the same technological result, namely, the difference of the concentrations of the highly volatile component and the liquid that enters the real tray and leaves it. Equating the right-hand sides of these expressions alternately with account for (10), (16), (22), and (28) makes it possible to obtain the following relations for the efficiencies of all the considered variants of mass transfer:

$$\frac{\dfrac{L}{mV} - 1 + \varphi}{E_{1\varphi}} = \frac{L}{mV} - 1 + \frac{\varphi}{E_{2\varphi}} = \frac{\dfrac{L}{mV} + \varphi}{E_{3\varphi}} - 1 = \frac{L}{mV} - \frac{1 - \varphi}{E_{4\varphi}}\,. \tag{29}$$

The relations obtained can be used to analyze and compare the considered variants of mass transfer in countercurrent motion of a vapor and a liquid on a tray.

With total mixing of a tray liquid (φ = 1), formulas (9), (15), (21), and (27) are transformed into the corresponding relations obtained for concurrent motion [7], but in the absence of mixing (φ = 0) they are transformed into the formulas obtained for the countercurrent motion of the interacting phases [8]. Thus, the above variants can be represented as general models, while mass transfer with counter- and concurrent motion of the components as their boundary cases.

Figures 2 and 3 show graphic representations of the interrelations of the tray efficiencies with different mixing and L/mV = 1.5.

In the real range of efficiencies for various degrees of mixing, $E_{1\varphi} > E_{2\varphi}$ (Fig. 2a). It should be noted that their difference decreases with increase in φ. The inverse ratio of the efficiencies occurs when their values are smaller than zero or larger than unity.

As is seen from Fig. 2b, the real values of $E_{3\varphi}$ are higher than $E_{1\varphi}$. As in the previous case, an increase in the mixing entails a decrease in the difference of the efficiencies considered but does not equalize them even at φ = 1. In unreal regions of efficiencies, when they are larger than unity or smaller than zero,

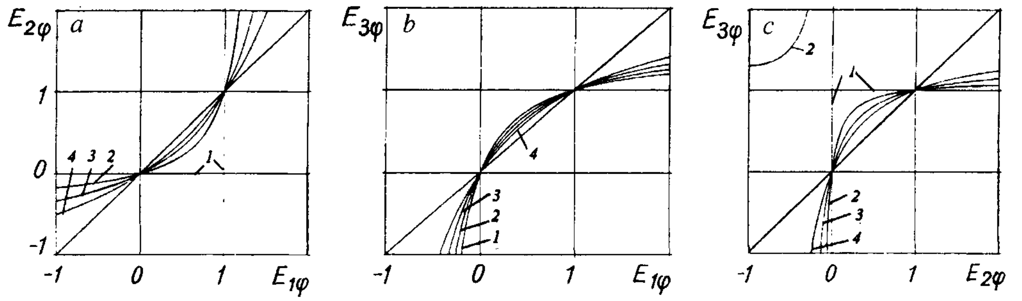

Fig. 2. Dependences of the efficiencies $E_{2\varphi}$ (a) and $E_{3\varphi}$ (b) on $E_{1\varphi}$ and $E_{3\varphi}$ on $E_{2\varphi}$ (c) at $L/mV = 1.5$ and different mixing: 1) $\varphi = 0$; 2) 0.2; 3) 0.5; 4) 1.0.

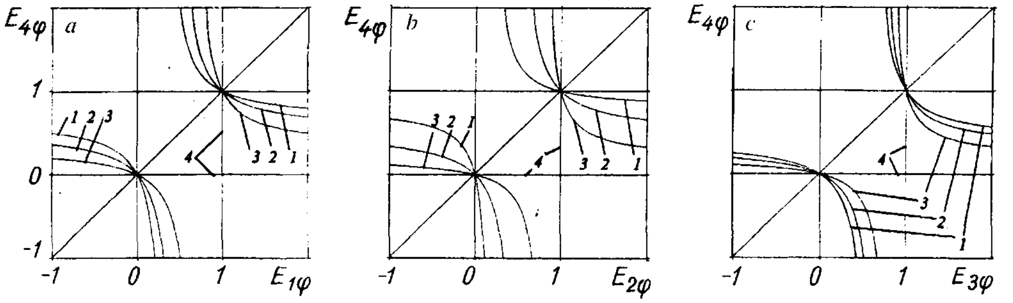

Fig. 3. Efficiency $E_{4\varphi}$ vs. $E_{1\varphi}$ (a), $E_{2\varphi}$ (b), and $E_{3\varphi}$ (c) at $L/mV = 1.5$ and different mixing: 1-4) same as in Fig. 2.

$E_{3\varphi} < E_{1\varphi}$. A comparison of Fig. 2a and b reveals that the ratio of $E_{1\varphi}$ and $E_{3\varphi}$ depends on the degree of mixing to a lesser extent than for the pair of $E_{1\varphi}$ and $E_{2\varphi}$.

The efficiency $E_{3\varphi}$ is higher than $E_{2\varphi}$ in the real range and lower at their unreal values (Fig. 2c). A comparison of curves a, b, and c in Fig. 2 shows that the mixing exerts an effect on the ratios of $E_{1\varphi}$ and $E_{2\varphi}$ and $E_{2\varphi}$ and $E_{3\varphi}$ to a greater extent and on $E_{1\varphi}$ and $E_{3\varphi}$ to a lesser extent.

Figure 3 shows that the real values of one efficiency correspond to unreal values of the other. Here, it can be noted that the mixing effect on these relations is at its maximum at $E_{1\varphi} > 1$ and $E_{4\varphi} < 0$ (Fig. 3a) when $E_{4\varphi} > 1$ and $E_{2\varphi} < 0$ (Fig. 3b) and at its minimum at $E_{4\varphi} > 1$ and $E_{3\varphi} < 0$ (Fig. 3c). Moreover, as a comparison of Figs. 2 and 3 reveals, the greater effect of φ in the graphs of the latter case is, apparently, attributable to the greater sensitivity of the fourth model of mass transfer to mixing.

The comparison made shows that mixing exerts a pronounced influence on the efficiency of mass transfer. The relations suggested can be used for interpretation of experimental and industrial data, in particular, in analyzing the mixing effect on the operation of rectification columns and in investigating the dependence of mixing on design and operational parameters.

In calculation procedures, use can be made of all the considered variants; however, preference should be given to the first three variants since the fourth model can yield unreal values of efficiency. It should be borne in mind that the first two models are less sensitive to mixing and their use is less fruitful as compared to the third variant.

NOTATION

E, tray efficiency; φ, degree of mixing; L, molar liquid flow; m, equilibrium coefficient; V, molar vapor flow; x, y, concentration of the highly volatile component in the liquid and vapor, respectively. Subscripts and superscripts: φ, parameter allowing for mixing; r, ideal displacement; liq, liquid phase; n, No. of the tray under consideration; $n - 1$, No. of the preceding tray in the direction of vapor motion; m, ideal mixing; v, vapor phase; *, equilibrium state; 1 – 4, Nos. of the considered variants of change in the concentrations.

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